

Dronacharya Group of Institutions, Gr. Noida
Department of Applied Sciences (First Year)
Even Semester (2020-2021)
Objective Question Bank

Subject Name & Code: Engineering Mathematics II (KAS 203T)

Unit-IV (COMPLEX VARIABLE-DIFFERENTIATION/Integration)

1. A continuous curve which does not have a point of self-intersection is called
(a) Simplecurve (b) Multiplecurve (c) Integralcurve (d) None
2. Simplecurve is also called
(a) Multiplecurve (b) Jordan curve (c) Integralcurve (d) None
3. In case of path of integral C is a closed curve then the integration $\int_C f(z) dz$ is called
(a) MultipleIntegral (b) Jordan Integral (c) Contour Integral (d) None
4. A region which is not simply connected is called ...region
(a) Multiplecurve (b) Jordanconnected
(c) Connectedcurve (d) Multi-connected
5. If $f(z)$ is analytic and $f'(z)$ is continuous at all points inside and on a simple closed curve C , then
a) $\int_C f(z) dz = 0$ b) $\int_C f(z) dz \neq 0$ c) $\int_C f(z) dz = 1$ d) $\int_C f(z) dz \neq 1$
6. If $f(z)$ is analytic and $f'(z)$ is continuous at all points in the region bounded by the simple closed curve C_1 and C_2 , then
a) $\oint_{C_1} f(z) dz = \oint_{C_2} f(z) dz$ b) $\oint_{C_1} f(z) dz \neq \oint_{C_2} f(z) dz$
c) $\oint_{C_1} f'(z) dz = \oint_{C_2} f'(z) dz$ d) $\oint_{C_1} f(z) dz \neq \oint_{C_2} f(z) dz$
7. A point z_0 at which a function $f(z)$ is not analytic is known as a.....of $f(z)$
a) Residue (b) Singularity (c) Integral (d) None
8. If the principal part contains an infinite number of non-zero terms of $(z - a)$ then $z = a$ is known as
a) Poles (b) Isolated Singularity (c) Essential Singularity (d) Removable Singularity
9. The singularity of $f(z) = \frac{z+3}{(z-1)(z-2)}$
a) $z=1,3$ b) $z=1,0$ c) $z=1,2$ d) $z=2,3$
10. A zero of an analytic function $f(z)$ is a value of z for which

- (a) $f(z) = 0$ b) $f(z) = 1$ c) $f(z) \neq 0$ d) $f(z) \neq 1$

11. Singularity of $f(z) = \frac{1}{1-e^z}$ at $z=2\pi i$

- a) Isolated singularity b) Non isolated singularity

- c) Simple pole d) none of these

12. Singularity of $f(z) = \frac{z - \sin z}{z^3}$ at $z=0$

2. a) Isolated essential singularity b) Non isolated essential singularity

3. c) Simple pole d) Removable singularity

13. Singularity of $f(z) = \frac{1}{e^{z-a}}$ at $z=a$

- a) Essential singularity b) Removable singularity c) Simple pole d) none of these

14. If the function $f(z)$ has a pole of order $m > 1$ at the point $z = a$, then $\text{Res } f(a) =$

a) $\frac{1}{m!} \lim_{z \rightarrow a} \left[\frac{d^m}{dz^m} \left\{ (z-a)^m f(z) \right\} \right]$

b) $\frac{1}{m-1!} \lim_{z \rightarrow a} \left[\frac{d^{m-1}}{dz^{m-1}} \left\{ (z-a)^m f(z) \right\} \right]$

c) $\frac{1}{m-1!} \lim_{z \rightarrow a} \left[\frac{d^{m-1}}{dz^{m-1}} \left\{ (z-a)^m f(z) \right\} \right]$

d) Zero

15. The function $\frac{1}{z(z-1)^3}$ has a pole of order p and residue r at $z = 1$, then.

- a) $p = 1, r = 1$ b) $p = 3, r = \frac{1}{3}$ c) $p = 3, r = 1$ d) $p = 1, r = 1$

16. For the function $f(z) = \frac{\sin \sqrt{z}}{\sqrt{z}}$, $z = 0$ is

- a) Essential singularity b) removable singularity c) pole d) none

17. The residue of $f(z) = \frac{1}{(z^2 + a^2)}$ at $z = ai$ is

- a) $\frac{1}{8!a^2}$ b) $\frac{1}{4!a^3}$ c) $\frac{1}{2!a^4}$ d) none

18. The residue of $f(z) = \cot z$ at its pole

- a) 1 b) 2 c) 0 d) -1

19. The residue of $f(z) = \frac{z^2}{(z^2 + 3z + 2)}$ at the pole $z = -1$

a) -4 **b) 1** c) 1/3 d) 3

20. The residue of $f(z) = \frac{z^2}{(z^2 + 3z + 2)^2}$ at the pole $z = -1$

a) **-4** b) 1 c) 1/3 d) 3

21. The residue of $f(z) = \frac{2z+1}{(z^2 - z - 2)}$ at the pole $z = -1$

a) -4 b) 1 **c) 1/3** d) 3

22. The value of the integral $\int_c \frac{z^2 - z + 1}{z - 1} dz$ where c is $|z| = 1/2$

a) $2\pi i$ b) $4\pi i$ **c) 0** d) $4\pi i/7$

23. The value of the integral $\int_c \frac{z^2 + 1}{z^2 - 1} dz$ where c is $|z - 1| = 1$

a) $2\pi i$ b) $4\pi i$ c) 0 d) $4\pi i/7$

24. The value of the integral $\int_c \frac{z}{(z-1)^2(z+2)} dz$ where c is $|z| = 3/2$

a) $2\pi i$ b) $4\pi i$ c) 0 **d) $4\pi i/7$**

25. The value of the integral $\int_c \frac{4 - 3z}{z(z-1)(z-2)} dz$ where c is $|z| = 3/2$

a) $2\pi i$ b) $4\pi i$ c) 0 d) $4\pi i/7$

26. The value of the integral $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where c is $|z| = 2$

a) $4\pi i/3e^2$ b) $2\pi i/3e^2$ c) 0 **d) $8\pi i/3e^2$**

27. The value of the integral $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where c is $|z| = 3$

a) $4\pi i$ b) $\pi/16$ c) 4π d) πi

28. The value of the integral $\int_c \frac{1}{z^2 + 4} dz$ where c is $|z - i| = 2$

a) $4\pi i$ **b) $\pi/16$** c) 4π d) πi

29. The value of the integral $\int_c \frac{1}{\sin z} dz$ where c is $|z| = 4$

a) $4\pi i$ **b) $2\pi i$** c) $-2\pi i$ d) $-4\pi i$

30. The value of the integral $\int_c \frac{e^z}{(z^2 + 1)} dz$ where c is $|z| = 1/2$

a) 1 b) -1 **c) 0** d) 2

31. If f have an isolated singularity at $z = a$ and $f(z) = \sum_{-\infty}^{\infty} a_n(z-a)^n$ is its Laurent's expansion about $z = a$, then

residue of $f(z)$ at $z = a$ is

- a) a_{-1} b) a_0 c) a_{-2} d) a_1

32. For the function $f(z) = e^{\frac{1}{z}}$, $Z = 0$ is

- a) An essential singularity b) removable singularity c) a pole d) None

33. At $Z = \infty$, the function $f(z) = \cos z - \sin z$ has

- a) removable singularity b) pole c) essential singularity d) None

34. Expansion of $\frac{1}{z-2}$ for $|z| > 2$

- a) $-\frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n$ b) $\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n$ c) $-\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n$ d) $\frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n$

35. Expansion of $\frac{1}{z-2} - \frac{1}{z-1}$ for $|z| < 1$

- a) $-\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n + \sum_{n=0}^{\infty} z^n$ b) $\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n + \sum_{n=0}^{\infty} z^n$
c) $-\frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n + \sum_{n=0}^{\infty} z^n$ d) $\frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n + \sum_{n=0}^{\infty} z^n$

36. Expansion of $\frac{1}{z-2} - \frac{1}{z-1}$ for $|z| > 2$

- a) $-\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n + \sum_{n=0}^{\infty} \frac{1}{z^n}$ b) $\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n + \sum_{n=0}^{\infty} \frac{1}{z^n}$
c) $\frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n - \sum_{n=0}^{\infty} \frac{1}{z^n}$ d) $-\frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n - \sum_{n=0}^{\infty} \frac{1}{z^n}$

37. Expansion of $\frac{1}{z-2} - \frac{1}{z-1}$ for $1 < |z| < 2$

- a) $-\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n - \frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n$ b) $\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n + \sum_{n=0}^{\infty} \frac{1}{z^n}$
c) $\frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n - \sum_{n=0}^{\infty} \frac{1}{z^n}$ d) $-\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n + \frac{1}{z} \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n$

38. The line integral of function $\mathbf{F} = yz\mathbf{i}$, in the counterclockwise direction, along the circle $x^2 + y^2 = 1$ at $z = 1$ is

- a) 4π b) 2π c) -2π d) π

39. Integration of the complex function $\int_c \frac{z^2}{z^2 - 1} dz$ in the counterclockwise direction, around $|z-1|=1$, is

- a) $-\pi i$ b) 0 c) πi d) $2\pi i$

40. $\int_c \frac{z^2 - 4}{z^2 + 4} dz$ Evaluated anticlockwise around the circle $|z-i|=2$

a) -4π

b) 0

c) $\pi+2$ d) $2+2i$

41. $f(z) = \frac{1}{z+1} - \frac{2}{z+3}$. If C is a counterclockwise path in the z -plane such that $|z+1|=1$, the value of $\frac{1}{2\pi i} \int_c f(z) dz$ is

a) -2

b) -1

c) 1

d) 2

42. If C is a circle of radius r and centre at a and oriented anticlockwise, then $\oint_c \frac{dz}{z-a}$

a) 2π b) $2\pi i$ c) πi

d) none of these

43. The domain $1 < |z| < 2$ is

a) Simply connected

b) doubly connected

(c) Multi-connected

(d) None of these

44. $\oint_{|z|=1} \sin z dz$ is

a) 2π b) $2\pi i$

c) 0

d) 2

45. The value of the integral $\int_c \frac{e^z}{(z)^3} dz$ where c is $|z|=1$

a) $2\pi i$ b) πi

c) 0

d) 2

46. $\oint_{|z|=1} \sec z dz$ is

a) 2π b) $2\pi i$

c) 0

d) 2

47. $\int_c \frac{1}{z^2 - 5z + 6} dz$ where C is the unit circle $|z|=1$

a) -2π b) πi

c) 0

d) none of these

48. Singularity of $f(z) = \sin \frac{1}{1-z}$ at $z=1$

a) Isolated singularity

b) Non isolated singularity

c) Simple pole

d) none of these

49. The value of the integral $\int_0^{1+i} z^2 dz$

a) $\frac{2}{3} + \frac{2}{3}i$ b) $-\frac{2}{3} + \frac{2}{3}i$ c) $\frac{4}{3} + \frac{2}{3}i$

d) none of these

50. The value of the integral $\int_c \frac{e^{iz}}{(z)^3} dz$ where c is $|z|=1$

a) $-\pi i$ b) πi

c) 0

d) 2

51. The value of the integral $\int_c \frac{1}{z^2 + 9} dz$ where c is $|z - 3i| = 4$

a) $-\pi/4$ b) $\pi/2$ c) $\pi/3$

d) 0

52. The value of the integral $\int_c \frac{\cos \pi z^2 + \sin \pi z^2}{(z+1)(z+2)} dz$ where c is $|z|=3$
- a) $4\pi i$ b) $2\pi i$ c) 4π d) $-4\pi i$
53. The value of the integral $\oint_c \frac{\cos z}{(z-\pi)} dz$ where c is $|z-1|=3$
- a) $-4\pi i$ b) $-2\pi i$ c) -4π d) πi
54. The value of the integral $\oint_c \frac{\tan z}{(z^2-1)} dz$ where c is $|z|=3/2$
- a) $-\pi i \tan 1$ b) $-4\pi i \tan 1$ c) $\pi i \tan 2$ d) $4\pi i \tan 1$
55. The value of the integral $\oint_c \frac{e^z}{(z^2+1)} dz$ where c is $|z-i|=1$
- a) $\pi(\cos 1 + i \sin 1)$ b) $2\pi(\cos 1 + i \sin 1)$ c) $4\pi(\cos 1 + i \sin 1)$ d) none of these
56. Let $f(z) = \frac{e^z}{(z-1)(z+3)^2}$ and C be the circle $|z|=3/2$ described in the positive sense. Then $\int_C f(z) dz$ is
- a) 0 b) $\frac{\pi e i}{8}$ c) $\frac{-\pi e i}{8}$ d) $\frac{\pi i (e - 5e^{-2})}{8}$
57. Given that 'a' lies inside C, the value of the integral $\frac{1}{2\pi} \int_C \frac{z - e^z}{(z-a)^3} dz$ is
- a) $e^a(1+a)$ b) $e^a \left(1 + \frac{a}{2}\right)$ c) e^a d) $e^a \left(\frac{1+a}{2}\right)$
58. $\int_{|z+1|=2} \frac{z^2}{4-z^2} dz =$
- a) 0 b) -2π c) 2π d) 1
59. When $0 < |z| < 4$, the expansion of $\frac{1}{4z-z^2}$ is
- a) $\sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n-1}}$ b) $\sum_{n=0}^{\infty} \frac{(-1)^n z^{n+1}}{4^{n+1}}$ c) $\sum_{n=0}^{\infty} \frac{z^{n+1}}{4^{n+1}}$ d) None
60. Residue of $\frac{z^3}{z^2-1}$ at $z=\infty$ is
- a) 1 b) -1 c) 0 d) ∞
61. Laurent's expansion of the function $\frac{1}{z^2-3z+2}$ for $|z| > 2$ is

a) $\sum_{n=0}^{\infty} \frac{2^{n-1}}{z^{n+1}}$ b) $\sum_{n=0}^{\infty} \frac{2^n}{z^{n+1}}$ c) $\sum_{n=0}^{\infty} \frac{2^{n+1}}{z^{n+1}}$ d) $\sum_{n=0}^{\infty} \frac{2^n}{z^n}$

62. Which of the following does give the residue at $z = \infty$ of any function $f(z)$ is/are

a) $\text{Res}(z = \infty) = -\frac{1}{z^2} f\left(\frac{1}{z}\right)$

b) $-\frac{1}{2\pi i} \int_C f(z) dz$

c) Negative of the coefficient of $\frac{1}{z}$ in the expansion of $f(z)$ in *nb* of zero.

d) All of the above.

63. The residue of $f(z)$ at $z = 2$, where $f(z) = \frac{e^{-z}}{(z-2)^4}$

a) $\frac{1}{6}$ b) $\frac{e^2}{6}$ c) $\frac{-1}{6e^2}$ d) $\frac{1}{6e^2}$

64. The expansion of $f(z) = \frac{1}{z(z^2 - 3z + 2)}$ for the region $|z| < 2$

a) $\frac{1}{2z} + \sum_{n=0}^{\infty} z^n - \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n$

b) $\frac{1}{2z} + \sum_{n=0}^{\infty} z^n + \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n$

c) $\frac{1}{2z} + \frac{1}{z} \sum_{n=0}^{\infty} z^n + \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n$

d) $\frac{1}{2z} + \frac{1}{z} \sum_{n=0}^{\infty} z^{-n} - \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{z}{2}\right)^n$

65. If a single value function $f(z)$ is not defined at $z = a$ but $\lim_{z \rightarrow a} f(z)$ exist, then $z = a$ is known as

a) In isolated singularity b) an essential singularity c) a removable singularity d) pole

66. Residue of $\frac{z^2 - 2z}{(z+1)^2(z^2 + 4)}$ at double pole is

a) $\frac{4}{25}$ b) $\frac{-4}{5}$ c) $\frac{-14}{25}$ d) $\frac{14}{25}$

67. The value $\int_C \frac{e^{3zi}}{(z+\pi)^3} dz$ where $C \equiv |z - \pi| = 3.2$, is

- (A) 3.1 (B) 0 (C) 2.5 (D) None of these.

68. The value $\oint_C \frac{\cos \pi z}{(z^2 - 1)} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$, is

- (A) -1 (B) 1 (C) 0 (D) None of these.

69. Taylor's Theorem is applicable in the

- (A) Circle only (B) Everywhere (C) Both A & B (D) None of these.

70. Laurent's Theorem is valid in

- (A) Annulus only (B) circle (C) Both A & B (D) None of these.

71. Expansion of $1/(z^2 - 3z + 2)$ in the region $1 < |z| < 2$, is

- (A) $\frac{-1}{2} \left(1 - \frac{z}{2}\right)^{-1} - \frac{1}{z} \left(1 - \frac{1}{z}\right)^{-1}$ (B) $-\frac{1}{2} \left(1 - \frac{z}{2}\right)^{-1} + (1 - z)^{-1}$ (C) Both A & B (D) None of these.

72. The limit points of zeros of an analytic function gives

- (A) An isolated essential singularity (B) poles (C) Non-isolated singularity (D) None of these.

73. The limit point of poles of an analytic function gives

- (A) An isolated essential singularity (B) poles (C) Non-isolated singularity (D) None of these.

74. This is residue of $\{f(z) \text{ at } z = a\} = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1} [(z-a)^m f(z)]}{dz^{m-1}}$.

- (A) pole of order m (B) pole of order m-1 (C) Both A & B (D) None of these.

75. The statement "if $f(z)$ is analytic function of z and if $f'(z)$ is continuous at each point within and on a closed contour C , then $\int_C f(z) dz = 0$ " known as by

- (A) Cauchy Integral theorem (B) Cauchy Integral Formula (C) Cauchy Residue theorem (D) None of these.

76. The statement "if $f(z)$ is analytic within and on a closed contour C and if a is any point within C , then

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} dz$$
 known as by.

- (A) Cauchy Integral theorem (B) Cauchy Integral Formula (C) Cauchy Residue theorem (D) None of these

77. The statement "if $f(z)$ analytic function at all points inside and on a simple closed curve C , except at a finite number of points isolated singular points within C , then $\oint_C f(z) dz = 2\pi i$ (sum of residues at singular points within C)" known as by

- (A) Cauchy Integral theorem (B) Cauchy Integral Formula (C) Cauchy Residue theorem (D) None of these

78. Using Residue theorem, evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where $C \equiv |z| = 5/2$.

- (A) $2\pi i$ (B) πi (C) $5/2$ (D) None of these.

79. Jordan's Inequality is

- (A) $\frac{2}{\pi} < \frac{\sin \theta}{\theta} < 1$ (B) $\frac{4}{\pi} < \frac{\sin \theta}{\theta} < 5$ (C) Both A & B (D) None of these.

80. Find the residue of $f(z) = \frac{2z+1}{z^2 - z - 2}$ at $z = -1$.

- (A) $1/3$ (B) $3/5$ (C) $-2/5$ (D) None of these.